Its the Economy Stupid: Predictive Theory of Belief Shift Connecting Economic Stress to Societal Polarization

David Yang[1,3], Jin Li[1,3], James Evans[2,4,6] and Ishanu Chattopadhyay,[1,4,5★]

[1]Department of Medicine, University of Chicago, Chicago, IL 60637, USA
[2]Department of Sociology, University of Chicago, Chicago, IL 60637, USA
[3]Department of Computer Science, University of Chicago, Chicago, IL 60637, USA
[4]Committee on Quantitative Methods in Social, Behavioral, and Health Sciences, University of Chicago, Chicago, IL 60637, USA
[5]Committee on Genetics, Genomics & Systems Biology, University of Chicago, Chicago, IL 60637, USA
[6]Santa Fe Institute, Santa Fe NM 87501, USA

★To whom correspondence should be addressed: e-mail: ishanu@uchicago.edu.

Human society is a complex system shaped by the beliefs of its inhabitants. Missing a reliable theory of belief evolution, we are limited in our understanding of how thought-centers mature, how societal polarization emerges, and how and if we can estimate individual and group-wise worldviews from partial information. However, despite significant interest, predictive models of opinion shift have proven elusive. In this study, we introduce the CogNet architecture – a computational framework to discover and operationalize dependencies between individual opinions on diverse topics, to ultimately craft a predictive theory of belief shift. Automatically analyzing raw survey data, the CogNet architecture can reason with, and predict, responses to thousands of topics, and is validated to be reliably predictive at the level of individual participants. We propose an emergent metric (q-distance) on the space of opinions quantifying world-view differences, where the q-distance between two sets of opinions is shown to scale as the log-likelihood of a spontaneous shift from one to another. Learning from data, the q-distance function automatically adapts to the fluid geometry of the evolving opinion space, yielding domain-agnostic measures of societal polarization. Analyzing responses from $> 60,000$ US participants over approximately half a century collected via the General Social Survey[1], we validate the CogNet framework by demonstrating that missing information in an individual's worldview may be reliably reconstructed. Investigation with the validated CogNet architecture suggests that the divide between ideological thought-centers is likely modulated by economic variables, *i.e.*, a faltering economy causally worsens polarization. Additionally, we discover that affective polarization is a likely causal precursor to ideological polarization, yielding a valuable insight into how social structures respond to economic stress: opinion clusters increase in numerosity and move apart, leading to widening gap between dominant ideologies. Such tractable tools to uncover opinion dynamics may help foster more effective socio-economic policy, and illuminate new frontiers in social theory.

THE recent emergence of effective disinformation campaigns to influence the political landscape, has highlighted the importance of understanding the mechanism of how opinions shift and coalesce across the social hierarchy. It is particularly important to understand the mechanisms that modulate societal polarization, both as a fundamental question of theory, and for designing more effective social policies. In this study we develop a computational framework that can reliably predict opinions from incomplete information, which then yields new validated tools for the measurement and analysis of evolving social structures (Fig. 1).

A highly polarized society is generally deemed unhealthy[2], with the population becoming progressively prejudicial towards opposing political groups (affective polarization), or adopting increasingly extreme positions on ideological and policy questions (ideological polarization). Attitudes and ideological differences are inherently difficult to quantify; thus devising simple, effective yet efficiently computable measures to track polarization is challenging. In effect, despite widespread interest[3–14], general claims regarding polarization and its measurement[15,16] have remained vague and academic, with unclear practical value in informing policy.

Attempts at "explaining" polarization through mechanistic models has had even more limited success, rarely using real observations to inform the assumed rules of belief shift. Imitation, influence from and communication with "social neighbors" are assumed to be modulating beliefs, despite the fact that key characteristics *e.g.* lack of global consensus in social systems, are often not well-replicated[15,16]. These models may be summarized as attempting to capture broad characteristic of opinion dynamics as network diffusion simulations[17,18], and indeed were never meant to predict opinions held by particular individuals on specific topics. Personalized information filtering[19] on social media, as a promoter of polarization, has also been investigated[20–23], with recent results demonstrating that increased exposure to opposing views might make polarization worse[24], perhaps suggesting underlying mechanisms that do not follow commonly supposed rules.

The approach developed in this study aims to address these issues by formulating an intrinsically meaningful metric for comparing and contrasting two distinct sets of opinions that be might be held by two individuals. This metric, referred to as the q-distance, recognizes that individuals and their opinions do not exist in the vacuum, but are embedded within a fluid background of social constraints and emergent dependencies. Thus, opinions on different social topics are almost never independent, and while some of these dependencies are easy to intuit, others have more subtle structure. We begin by formulating an automated approach to computationally distill these dependencies from survey data, which leads to a detailed model realized as a recursive forest of conditional inference trees[25], which we call the Q-net (Fig. 2). In this study, this inference is demonstrated on the General Social Survey database, which provides a rich window into the opinions and beliefs held by the US populace over nearly half a century (1972-2021), documenting 31,670,949 responses to 6,209 unique query items from n=65,784 US residents. The year-specific Q-nets inferred from the GSS data informs us on how to appropriately measure deviations in opinions, modulating the corresponding q-distance function mapping pairs of *opinion vectors* to a positive number (the q-distance) between the opinion vectors. Syntactically, the Q-net comprises a set (forest) of decision trees, which are constructed with statistically significant node splits (conditional inference trees). For a given set of query items, we learn a distinct tree to predict each of these query items, where the remaining items act as features. Thus, the features in one tree provide the target labels in other trees, resulting in a recursive forest structure, which represents a detailed and nearly assumption-free model of conditional cross-dependencies between responses to different query items (Fig. 1). te

Thus, within our framework, individuals are represented by their opinion vectors, which can have missing responses. To compute the distance between two individuals $i.e.$ their possibly partially populated opinion vectors, it is not sufficient to simply specify their opinions, we must also note the time at which these opinions are recorded. And as the social background evolves over time, we can demonstrate that the distance between two fixed set of responses to a fixed set of query items ($i.e.$ two fixed opinion vectors) can vary solely because of the time-dependence of the inferred Q-nets. Thus, the distance between two opinion vectors can change if either the opinions change, or if only the social norms, beliefs and environment evolves around fixed responses, or both.

While arguably numerous definitions of legitimate "distance"s are conceivable, the q-distance between two opinion vectors is canonical in that it is simplest function that is a bona fide distance metric, and provably scales as the log-likelihood of a spontaneous opinion shift from one vector to the other. This information-theoretic property, established explicitly, induces a range of intrinsic, yet efficiently computable measures of polarization, which in turn uncovers key insights into the mechanics of polarization in the US society.

Indeed if we can measure the variation of the distance between fixed opinion vectors, then we can arguably measure ideological polarization by computing the distance between the extreme opinions or the poles. In the context of the US society we consider two poles, namely the ultra conservative and the ultra liberal response sets to a fixed set of socially contentious query items (Table I). The time-dependent distance between these polar vectors, referred to as the polar separation, then measures the log-likelihood of spontaneous change of one to another, and thus, larger this distance, harder it is to "bridge" the divide, and worse is the degree of ideological polarization.

We also consider a second measure of ideological polarization, referred to as the *embedding diameter*. Unlike the polar separation, which measures the distance between the fixed poles, the embedding diameter is an estimate of the distance between the further opinion vectors observed at a given time within a sub-sample of the year-specific set of participants. Thus, while the polar separation measures the distance between a theoretical pair of extreme opinions, the diameter measures the distance between observed pairs of extreme opinions. We also consider two measures of affective polarization, namely the *optimal number of clusters*, and the *average cluster separation* observed in the time-dependent metric embedding computed on basis of the q-distance (Fig. 2). Importantly, while ideological polarization measures quantify how removed the ideological positions are from each other, the affective polarization measures estimate the in-between distance and numerousity of emergent societal groups.

How can we validate the Q-net construction to establish that the q-distance does indeed capture a meaningful social distance? We leverage the generative property of the Q-net, which allows us to impute missing responses in a partial opinion vector. We randomly mask out-of-sample responses of a participant (participant not used in Q-net construction), and then reconstruct the missing values using the Q-net from the time period that the participant belongs to. Our results show that this reconstruction can be achieved reliably with a small probability

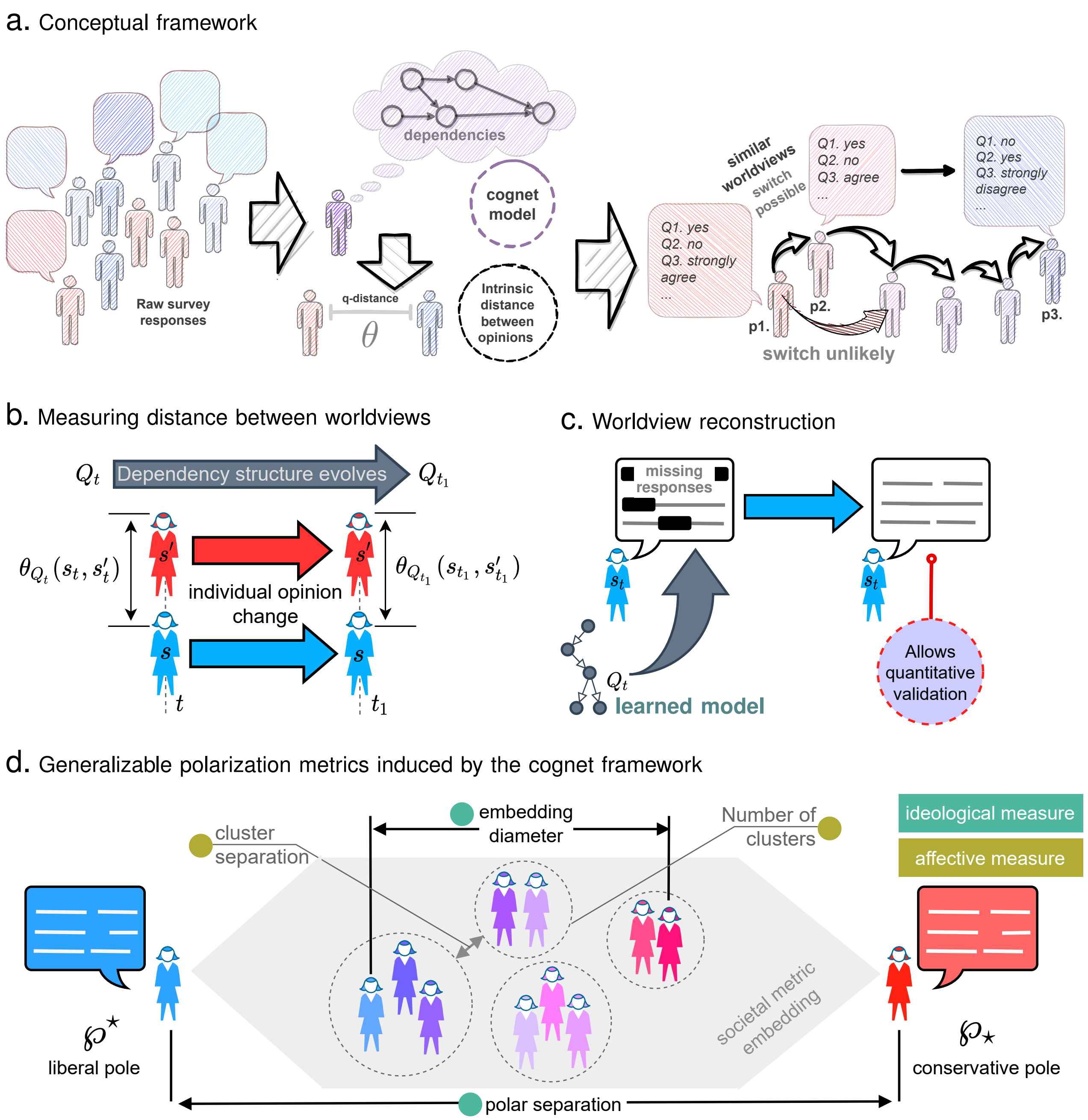


Fig. 1. **Conceptual framework.** Panel a illustrates that the CogNet architecture infers a model of dependencies between opinions, beliefs and demographic variables from raw survey data, without any apriori assumptions on the structure of such inferred reltionships. These dependencies are inferred as a recursive forest of conditional inference trees, known as a Q-net. The inferred Q-net induces an distance between any two possibly partially populated opinion vectors, and can be shown to scale approximately as the log-likelihood of spontaneous transition between two distinct belief vectors, thus making the q-distance a naturally meaningful metric on the space of opinions. Panel b illustrates that we can infer a Q-net specific to different time-periods, $e.g.$ year of a GSS survey, implying we can measure how the distance between two fixed opinion vectors vary over time, as the social environment changes. Panel c illustrates the idea that we can use the Q-net to estimate missing data on an individual's position of specific issues, thus probabilistically completing partially observed worldviews. We leverage this ability to validate the CogNet framework. Panel d illustrates four distinct measures of polarization that arise in the CogNet framework: two of these (the embedding diameter and the polar separation) are measures of ideological polarization, whereas the remaining two (number and spacing of clusters) are measures of affective polarization. Note "polar separation" is simply the q-distance between two extreme opinion vectors (the poles) to a fixed set of socially contentious questions (See Table I). Variation of the metric over time implies that the polar separation varies over time, although the poles themselves are held constant.

of error for out-of-sample participants ($< 10\%$), demonstrating simultaneously the validity of our framework for probing polarization , and for estimating worldviews of individuals from incomplete information (Fig. 3a-b, and Extended Data Tables I-VI). We noted that it is slightly harder to reconstruct opinions for the section of population with more extreme beliefs, $e.g.$, for the left and the right fringe defined as the set of participants with absolute

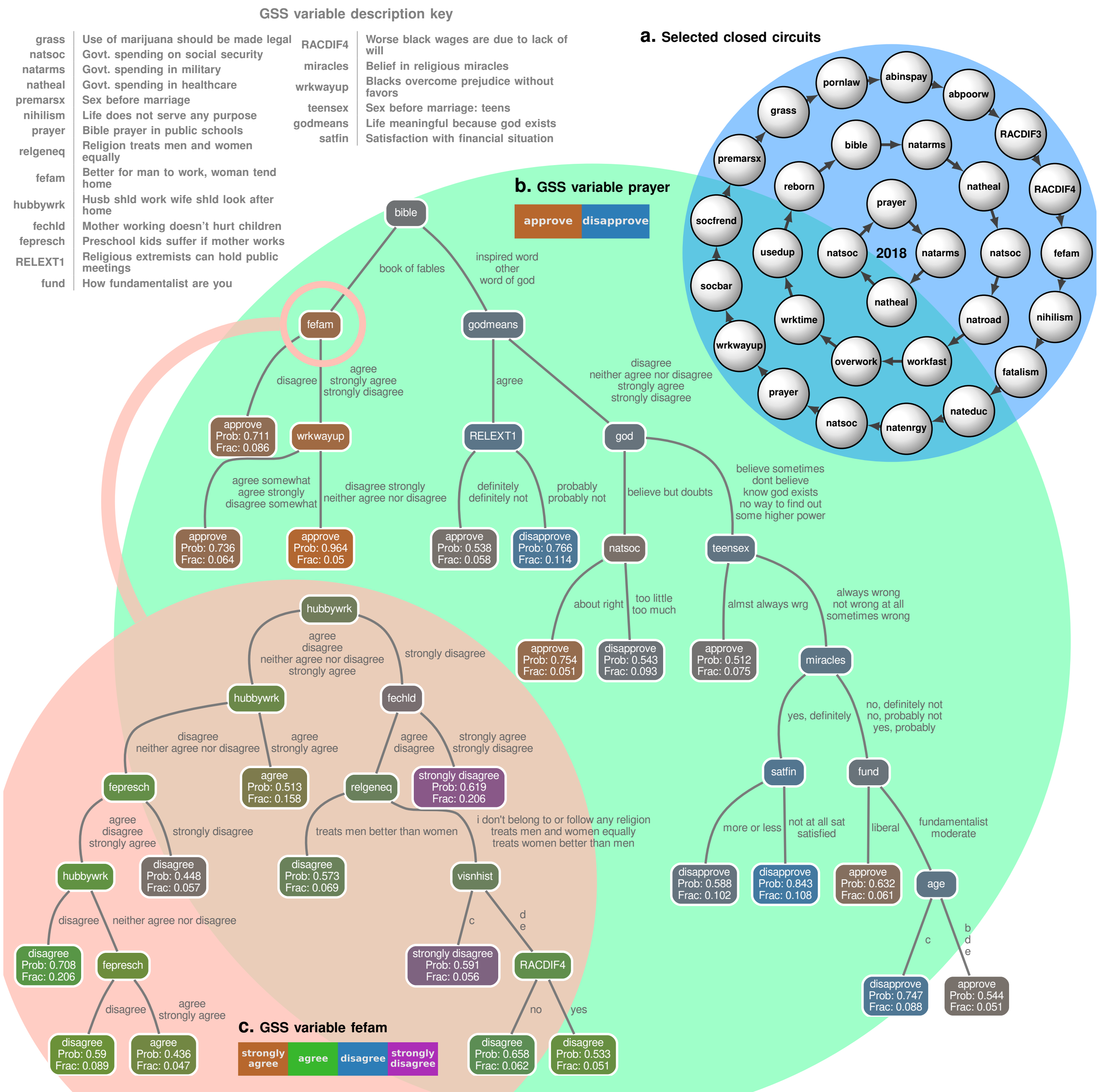


Fig. 2. **Q-net dependency framework.** Panel a illustrates some inferred dependencies in the Q-net inferred for the 2018 GSS survey. More specifically, we illustrate a selected set of closed shortest-path circuits in among the GSS variables, showing the interplay of social, political, demographic and educational backgrounds. Panel b and c illustrate two specific conditional inference trees inferred for the GSS variable **prayer** (support of bible prayer in public schools) and **fefam** (It is better for men to work and women tend home) respectively. Note that that these variables may be predicted using these trees using responses to other GSS variables as features, and the variables that act as features in these trees, are predicted by their own inference trees. For example, the **prayer** tree (panel b) uses **fefam** (panel c) as a feature. The descriptions of the GSS variables used in these two trees are shown in top left inset. Node colors correspond to the response distribution characterized by that node: colors of the "pure" responses (*e.g.* purely "approve" or purely "disapprove" in panel b) are shown under the panel titles. Since the nodes have a non-degenerate distribution over possible responses, the actual node color is a mixture of the colors of the pure responses. In the terminal nodes, "Prob." refres to the probability of the chosen decision, and "Frac." denotes the probability of ending up in that leaf.

ideology index > 0.7, the reconstruction error fails to be reduced for approximately 12% of the samples (although the average reduction in error is higher). For further validation, we also investigated if the CogNet framework can predict individual voting in the 2016 US Presidential election. We tested two approaches: 1) direct reconstruction of masked response to the GSS variable PRES16 (enumerating the candidate voted for), and alternatively 2) first localizing the participant in a race/gender opinion plane, and then using these coordinates as features in a standard machine learning classifier. We achieve out of sample AUC exceeding 84% and 90% respectively, significantly outperforming standard ML models using the raw responses as features (Fig. 3, panels c-e).

TABLE I
POLAR RESPONSE VECTORS ON A FIXED SET OF SOCIALLY CONTENTIOUS TOPICS*

index	description	conservative pole ($\wp_\star$)	liberal pole ($\wp^\star$)
abany	abortion should be legal if mother wants it for any reason	no	yes
abdefctw	abortion is wrong if there is a strong chance of serious defect in the baby	always wrong	not wrong at all
abdefect	abortion should be legal if there is a strong chance of serious defect in baby	no	yes
abhlth	abortion should be legal if mother's own health is seriously endangered by the pregnancy	no	yes
abnomore	abortion legal if mother does not want any more children	no	yes
abpoor	abortion should be legal if family has a very low income and cannot afford any more children	no	yes
abpoorw	wrong for woman to get abortion if low income	always wrong	not wrong at all
abrape	abortion should be legal if mother pregnant by rape	no	yes
absingle	abortion should be legal if mother is not married and does not want to marry the man	no	yes
bible	the bible is the actual word of god and is to be taken literally or is a book of fables	inspired word	book of fables
colcom	communist allowed to teach in a college	fired	not fired
colmil	militarists be allowed to teach in a college or university	not fired	not allowed
comfort	practicing a religion helps people to gain comfort in times of trouble and sorrow	strongly agree	strongly disagree
confed	confidence in federal government	hardly any	a great deal
conlabor	confidence in organized labor	hardly any	a great deal
godchnge	which best describes your beliefs about god	believe now, always have	don't believe now, never have
grass	use of marijuana should be made legal	not legal	legal
gunlaw	require a person to obtain a police permit before he or she could buy a gun	oppose	favor
intmil	interest in issues about military and defense policy	very interested	not at all interested
libcom	communist books allowed in your public library	remove	not remove
libhomo	book in favor of homosexuality allowed in public library	remove	not remove
libmil	allow militarists book in library	not remove	remove
libmslm	allow anti-american muslim clergymen's books in library	remove	not remove
maboygrl	mother's gene decides whether the baby is a boy or a girl	true	false
natarms	govt spending on military	about right	too much
natenvir	govt spending on environment	too much	too little
natfare	govt spending on welfare	too much	too little
natsoc	govt spending on social security	too much	too little
owngun	have in your home any guns or revolvers	yes	no
pillok	birth control should be available to teenagers between the agesof 14 and 16 if their parents do not approve	strongly disagree	strongly agree
pilloky	birth control to teenagers 14-16	strongly disagree	strongly agree
polabuse	policeman can strike a citizen who says vulgar and obscene things to the policeman	no	yes
pray	about how often do you pray	several times a day	never
prayer	Bible prayer in public schools	disapprove	approve
prayfreq	about how often do you pray	several times a day	never
religcon	religions bring more conflict than peace	strongly disagree	strongly agree
religint	people with very strong religious beliefs are often too intolerant of others	strongly disagree	strongly agree
reliten	would you call yourself a strong religious person	strong	no religion
rowngun	own a gun	yes	no
shotgun	own a shotgun	yes	no
spkcom	communist allowed to make a speech in your community	not allowed	allowed
spkmil	militarists allowed to make a speech in your community	allowed	not allowed
taxrich	describe taxes in america today	about right	much too low
viruses	antibiotics kill viruses as well as bacteria	definitely true	definitely not true

* No actual respondent is expected to align perfectly with these poles. However, a conservative is expected to lean towards the conservative pole and vice versa, $i.e$, the ideology index (Def. 8) for a conservative is expected to be negative.

With the validated framework in place, we focused on uncovering the driving mechanics of polarization in the US society, emergent over the past 50 years. Our results suggest that 1) economic variables are possibly strongly associated with polarization, and 2) that there is statistical evidence of a causal chain starting from economic variables, to affective polarization and finally to ideological polarization (Fig. 4). In Fig. 4a-b we plot the variation of the ideological and affective polarization measures respectively, showing that affective polarization

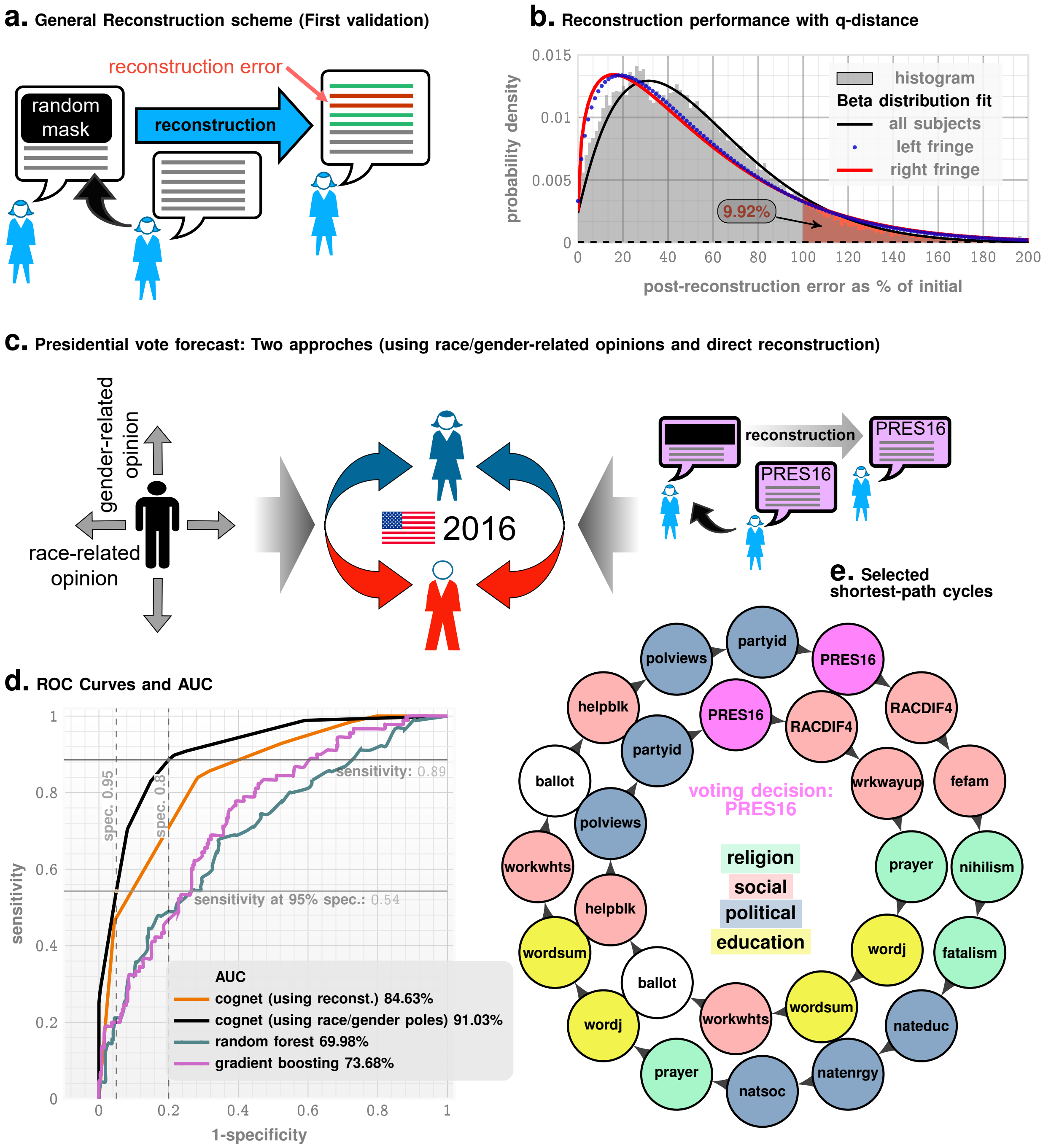


Fig. 3. **Missing opinion reconstruction and CogNet validation.** We mask off opinions for a randomly chosen 50% of the available responses in the out-of-sample participants within the polar items shown in Table I (panel a), and reconstruct them using q-sampling as described in the Methods. The results are shown in panel b, where we show the distribution of the post-reconstruction error (measured by the q-distance between the estimated opinion vector and the ground-truth), as a fraction of the pre-reconstruction error. Any result less than 100% is an improvement, with error $> 100\%$ indicates that our reconstruction did not succesfully improve the assessment of the masked opinions. We note that we can reduce the error in $> 90\%$ of the participants. It is somewhat easier to reconstruct extreme opinions on both ends of the belief spectrum (illustrated by the peaks of the left/right fringe, defined by an abolute ideology index > 0.7 occurring on the left of the scenarion where we consider all participants). Extended Data Tables I, II, III, IV, V and VI show examples of actual reconstruction, comparing the ground-truth responses with estimated ones in randomly chosen participants, with Table VI showing an example where the reconstruction was not very successful. The probability of such poor reconstruction is small, as shown in panel b. We also test if we can forecast individual voting in the 2016 presidential election (GSS variable PRES16), using either a selected set of variables to localize subjects in a race/gender-related opinion plane (See Extended Data Table VII), or reconstruction of the masked response. Panel d shows that the ROC curves, demonstrating that we achieve out-of-sample AUC¿90% beating out standard ML models using responses as features. Panel e illustrates two shortest path cycles involving the target GSS variable, showing the dependency across social, political, educational and religious beliefs.

minima precedes that for ideological polarization. Using three different lines of reasoning we then elucidate the dynamical connection between putative economic and political drivers of these social effects. In panel d we fit

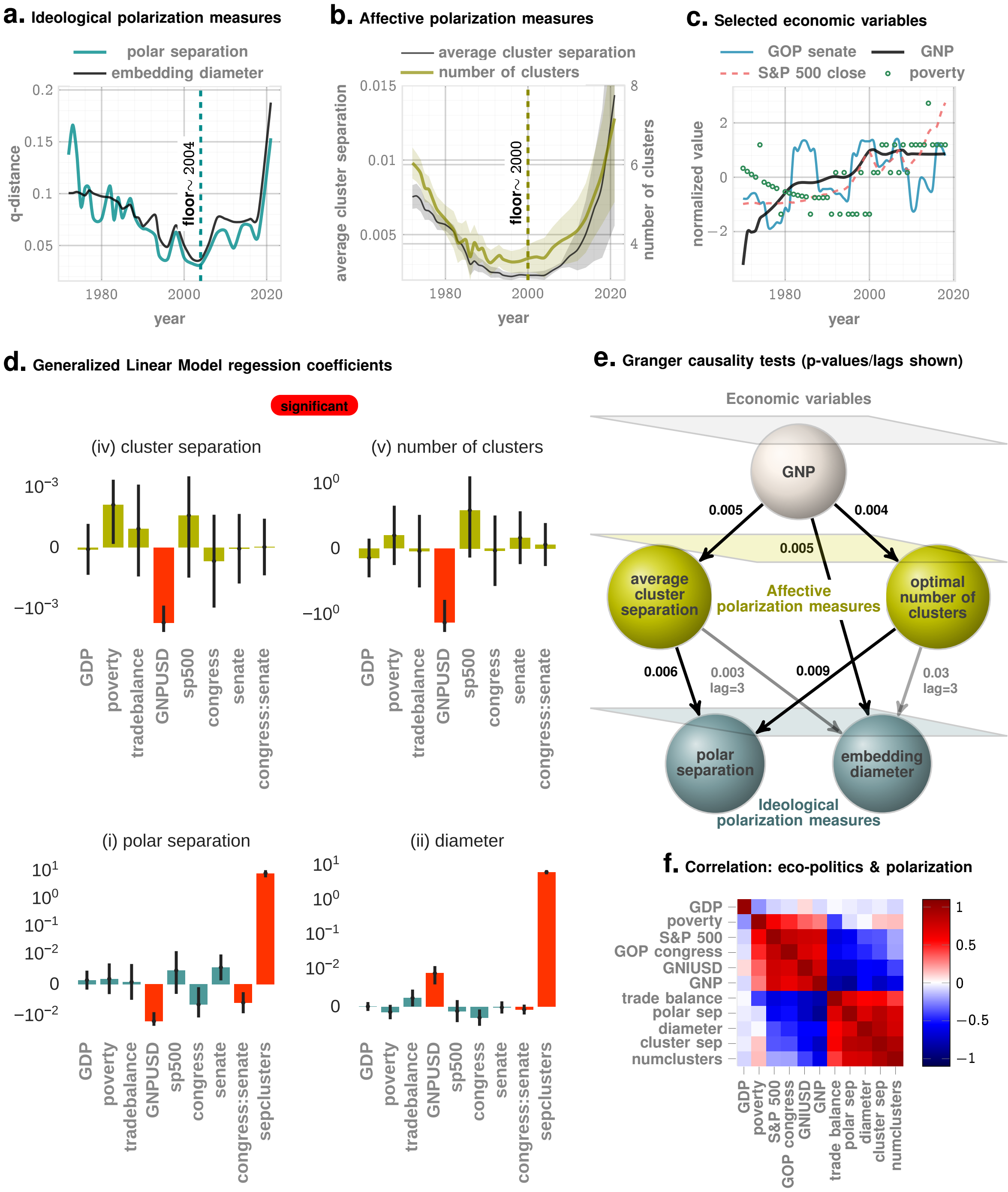


Fig. 4. **Polarization measures and link to GNP.** Panel a shows the variation in the measures of ideological polarization over time, which achieves a minimum approximately around 2004. Panel b illustrates the variation in affective measures of polarization, which achieve a minimum between approximately 1996-2000. Panel c plots a subset of key economic variables considered in this study, namely GNP, fraction of GOP senate representation, end-of-year S&P 500 close prices, and US Census Bureau's standard index of national poverty. Panel d shows GLM modeling with polarization measures as response variables (subpanels i-iv), in which GNP is the only significant variable for affective polarization, and GNP along with affective polarization are significant covariates for ideological polarization. Panel e carries out a standard Granger causality analysis, presenting only the statistically significant relationships, which suggest a causal chain **GNP** $\rightarrow$ **affective polarization** $\rightarrow$ **ideological polarization**. Panel f computes Pearson's correlation between the relevant variables, which corroborates the emerging statistically significant picture: GNP changes are associated with changes in societal polarization.

out a generalized linear model to the polarization response variables, and find Gross National Product (GNP) as the only significant factor for affective polarization. For ideological polarization, we find that in addition to GNP, affective polarization is also a significant contributor along with GOP senate representation over time. The regression model structures these analysis were chosen to maximize standard measures of goodness-of-fit including Bayesian and Akaike Information criterion (AIC). This particular observation is corroborated in a standard Granger causal analysis[26] (panel e), which shows all unidirectional significant causal links found amongst the set of all possible pairwise variables, which suggests that economic variables (GNP) $\rightarrow$ affective polarization $\rightarrow$ ideological polarization. Finally the Pearson's correlation matrix (panel f) indicates that the measures of polarization cluster together, as does the putative eco-political drivers, with the "bridge" between the clusters dominated by GNP, S&P500 close prices, and GOP senate representation.

Understanding societal polarization and its drivers is emerging as a fundamental challenge in policy making and governance, without which we risk irreversible erosion of democratic norms and institutions[27–29]. Our insight into polarization mechanics is not without precedence, and is well-reflected in the contemporary literature, where polarization, and particularly affective polarization, has been connected to rising inequality and economic decline[30]. And, similar conclusions are reached from comparative analysis of societal polarization emerging globally[31]. While the threat of growing affective polarization has been well recognized, the connection to ideological divide has remained more ambiguous[32]. That eco-political factors drive affective polarization, and not directly ideological separation, has also been suggested[4] before.

Thus, the individual components of the mechanics suggested here are less than surprising. However, we have taken a novel approach aimed at complementing the academic discourse: instead of devising simple models manually designed to reflect important perceived characteristics of polarization dynamics, or using proxies to measure polarization and then elucidate its correlation to eco-political variables, we have modeled the complex distribution of opinions across the US society at the level of the individual directly – learning highly complex structures that have shaped and constrained the shared belief space over half a century in the US. This analysis, relatively free from interpretive biases, provides purely quantitative and actionable evidence of the causal chain described above, and hopefully lays the foundation for machine inference to inform policy decisions.

METHODS

Data Sources

Our data is the complete GSS database procured from the National Opinion Research Center (NORC) at the University of Chicago. As mentioned, the survey has sampled $> 65K$ US residents over nearly half a century. We use 80% of the data for training, and the rest for out-of-sample validation. Data for the putative economic and political factors are obtained from the United States Census Bureau.

Basic Definitions and Notation

Definition 1 (Item Set)**.** *Let $\mathbb{I}$ be a finite set of questions (items) asked to a population of individuals. We call this the item set. Each item response can be either categorical, ordinal or real valued. The range of each item $i \in \mathbb{I}$ is denoted as Σ_i.*

Note that each respondent can be imagined to produce a single data point in a very high dimensional space, $e.g.$ if there ate 6000 items, then each set of $\leqq 6000$ responses from an individual is a point in a 6000 dimensional space. More importantly, perhaps, these items are not independent, and have non-trivial, and often surprising or counter-intuitive dependencies, which cannot be charted out a priori. We can think of $\mathbb{I}$ as the index set of a set of random variables, $i.e.$, the item $i \in \mathbb{I}$ indexes a random variable X_i taking values in Σ_i. Note that these random variables are not independent, and our task here is to infer these dependencies.

Definition 2 (Response Set or Sample)**.** *Given an item set $\mathbb{I}$, a response set or a sample is a set of responses to a subset of items in $\mathbb{I}$ from a specific individual. We allow partial responses, $i.e.$., a response set can only contain responses to any subset of $\mathbb{I}$.*

Definition 3 (Q-net Φ^P)**.** *The construction of the recursive decision forest, as a collection of conditional inference trees referred to as the Q-net, may be summarized as: If we have n questions/topics $X_1, \cdots, X_n$, and we have a subject responding with $x_{-i} \triangleq \{x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{n-1}, x_n\}$, then the distribution of responses to question X_i is given by $\Phi_i : \prod_{j \neq i} \Sigma_j \rightarrow \mathcal{D}(\Sigma_i)$ where $\mathcal{D}(\Sigma_i)$ is the set of all possible distributions over the set of all possible responses Σ_i. The Q-net Φ^P is the collection of all such decision trees computed on $\mathbb{I}$ for the participant population P.*

In this study we use conditional inference trees[25], to infer the component decision trees in a Q-net. In contrast to decision tree construction algorithms that perform univariate splits and use information measures such as the gini coefficient to select covariates, conditional inference trees uses multiple significance tests at each split, thus substantially resiting overfits.

Definition 4 (q-distance)**.** *For two opinion vectors x, y, our intrinsic metric (q-distance) is defined as:*

$$\theta_{P,Q}(x,y) \triangleq \mathbf{E}_i \left(\mathbb{J}^{\frac{1}{2}} \left(\Phi_i^P(x_{-i}), \Phi_i^Q(y_{-i}) \right) \right)$$

where P, Q are possibly two distinct populations with distinct Q-nets, such that $x \in P, y \in Q$ and $\mathbb{J}(\cdot,\cdot)$ is the Jensen-Shannon (JS) divergence[33]

If the populations are identical, we denote q-distance between x, y as $\theta_P(x,y)$ or $\theta(x,y)$ if the populations are clear from context.

Importantly, the square-root in the definition arises naturally from the bounds we are able to prove, and is dictated by the form of Pinsker's inequality[34], making sure that distances along a sequence of successive opinion vectors sum linearly. Since the JS divergence is a bona fide metric, and sums and scaling preserves metric properties, the q-distance satisfies the required properties of being a distance metric, with the exception of the requirement of being 0 if and only if the opinion vectors are identical. Thus, the q-distance is technically a pseudo-metric since distinct opinion vectors can induce the same distributions over each index, and thus evaluate to have a zero distance. This is actually desirable, since we do not want our distance to be sensitive to changes that are not socially relevant. The intuition is that not all opinion variations are equally important or likely.

We can extend the definition of q-distance to define a distance between an individual and a group (a sub-population), or between two groups, as follows:

Definition 5 (Pseudo-metric Between Individuals and Groups, and Two Groups)**.** *Using Hausdorff metric between sets:*

$$\forall x \in P, y \in Q,$$

$$\theta(x,Q) = \min_{y \in Q} \theta(x,y) \tag{1}$$

$$\theta(P,Q) = \max \left\{ \max_{x \in P} \theta(x,Q), \max_{y \in Q} \theta(y,P) \right\} \tag{2}$$

Estimating Goodness of Fit

For our modeling to be reliable, we need a quantitative test of how well the Q-net represents the survey data. Here, we formulate an explicit model membership test to address this.

Definition 6 (Membership Probability of an opinion vector)**.** *Given a population P inducing the Q-net Φ^P and an opinion vector x, we can compute the membership probability of x in the set of samples modeled well by the Q-net:*

$$\omega_x^P \triangleq Pr(x \in P) = \prod_{j=1}^{N} \left(\Phi_j^P(x_{-j})|_{x_j} \right) \tag{3}$$

which represents the probability that the Q-net generates the sample x.

Note that x_j is the j^{th} entry in x, and is thus an element in the set Σ_j. Since we are mostly concerned with the case where Σ_j is a finite set, $\Phi_j^P(x_{-j})|_{x_j}$ is the entry in the probability mass function corresponding to the element of Σ_j which appears at the j^{th} index in sequence x. We can assess the goodness of fit of an inferred Q-net by testing if the null hypothesis H_0: "samples have a higher probability of being generated by randomly selecting responses, compared to being generated by the inferred Q-net" is rejected. We find that for all years H_0 is rejected at $> 99.99\%$ significance level (Extended Data Fig. 4).

Theoretical Probability Bounds

The Q-net framework allows us to rigorously compute bounds on the probability of a spontaneous change of one opinion vector to another, brought about by chance variations. Not all perturbations in an opinion vector are likely or sociologically meaningful, $i.e.$, opinions of some topics are more likely to vary given the rest of one's opinions or beliefs. With the exponentially exploding number of possibilities in which an opinion vector over a large set of query items can vary, it is computationally intractable to exhaustively model this dynamics. Nevertheless, we can constrain the possibilities using the patterns distilled by the Q-net construction. We show in Theorem 1 that at a significance level α, with the number of query items N, the probability of spontaneous jump of an opinion vector x from population P to an opinion vector y in population Q, $Pr(x \rightarrow y)$ is bounded:

$$\omega_y^Q e^{\frac{\sqrt{8}N^2}{1-\alpha}\theta(x,y)} \geqq Pr(x \rightarrow y) \geqq \omega_y^Q e^{-\frac{\sqrt{8}N^2}{1-\alpha}\theta(x,y)} \tag{4}$$

where ω_y^Q is the membership probability of opinion vector y in Q.

Theorem 1 (Probability Bound)**.** *Given an opinion vector x of length N that transitions to $y \in Q$, we have the* *following bounds at significance level α.*

$$\omega_y^Q e^{\frac{\sqrt{8}N^2}{1-\alpha}\theta(x,y)} \geqq Pr(x \rightarrow y) \geqq \omega_y^Q e^{-\frac{\sqrt{8}N^2}{1-\alpha}\theta(x,y)} \tag{5}$$

where ω_y^Q is the membership probability of y in the population Q (See Def. 6), and $\theta(x,y)$ is the q-distance between x, y (See Def. 4).

Proof. Using Sanov's theorem[34] on large deviations, we conclude that the probability of spontaneous jump from $x \in P$ to $y \in Q$, with the possibility $P \neq Q$, is given by:

$$Pr(x \rightarrow y) = \prod_{i=1}^{N} \left(\Phi_i^P(x_{-i})|_{y_i} \right) \tag{6}$$

Writing the factors on the right hand side as:

$$\Phi_i^P(x_{-i})|_{y_i} = \Phi_i^Q(y_{-i})|_{y_i} \left(\frac{\Phi_i^P(x_{-i})|_{y_i}}{\Phi_i^Q(y_{-i})|_{y_i}} \right) \tag{7}$$

we note that $\Phi_i^P(x_{-i})$, $\Phi_i^Q(y_{-i})$ are distributions on the same index i, and hence:

$$|\Phi_i^P(x_{-i})_{y_i} - \Phi_i^Q(y_{-i})_{y_i}| \leqq \sum_{y_i \in \Sigma_i} |\Phi_i^P(x_{-i})_{y_i} - \Phi_i^Q(y_{-i})_{y_i}| \tag{8}$$

Using a standard refinement of Pinsker's inequality[35], and the relationship of Jensen-Shannon divergence with total variation, we get:

$$\theta_i \geqq \frac{1}{8}|\Phi_i^P(x_{-i})_{y_i} - \Phi_i^Q(y_{-i})_{y_i}|^2 \Rightarrow \left| 1 - \frac{\Phi_i^Q(y_{-i})_{y_i}}{\Phi_i^P(x_{-i})_{y_i}} \right| \leqq \frac{1}{a_0}\sqrt{8\theta_i} \tag{9}$$

where a_0 is the smallest non-zero probability value of generating the entry at any index. We will see that this parameter is related to statistical significance of our bounds. First, we can formulate a lower bound as follows:

$$\log \left(\prod_{i=1}^{N} \frac{\Phi_i^P(x_{-i})|_{y_i}}{\Phi_i^Q(y_{-i})|_{y_i}} \right) = \sum_i \log \left(\frac{\Phi_i^P(x_{-i})|_{y_i}}{\Phi_i^Q(y_{-i})|_{y_i}} \right) \geqq \sum_i \left(1 - \frac{\Phi_i^Q(y_{-i})_{y_i}}{\Phi_i^P(x_{-i})_{y_i}} \right) \geqq \frac{\sqrt{8}}{a_0} \sum_i \theta_i^{1/2} = -\frac{\sqrt{8}N}{a_0}\theta \tag{10}$$

Similarly, the upper bound may be derived as:

$$\log \left(\prod_{i=1}^{N} \frac{\Phi_i^P(x_{-i})|_{y_i}}{\Phi_i^Q(y_{-i})|_{y_i}} \right) = \sum_i \log \left(\frac{\Phi_i^P(x_{-i})|_{y_i}}{\Phi_i^Q(y_{-i})|_{y_i}} \right) \leqq \sum_i \left(\frac{\Phi_i^Q(y_{-i})_{y_i}}{\Phi_i^P(x_{-i})_{y_i}} - 1 \right) \leqq \frac{\sqrt{8}N}{a_0}\theta \tag{11}$$

Combining Eqs. 10 and 11, we conclude:

$$\omega_y^Q e^{\frac{\sqrt{8}N}{a_0}\theta} \geqq Pr(x \rightarrow y) \geqq \omega_y^Q e^{-\frac{\sqrt{8}N}{a_0}} \tag{12}$$

Now, interpreting a_0 as the probability of generating an unlikely event below our desired threshold ($i.e.$ a "failure"), we note that the probability of generating at least one such event is given by $1-(1-a_0)^N$. Hence if α is the pre-specified significance level, we have for $N >> 1$:

$$a_0 \approx (1-\alpha)/N \tag{13}$$

Hence, we conclude, that at significance level $\geqq \alpha$, we have the bounds:

$$\omega_y^Q e^{\frac{\sqrt{8}N^2}{1-\alpha}\theta} \geqq Pr(x \rightarrow y) \geqq \omega_y^Q e^{-\frac{\sqrt{8}N^2}{1-\alpha}\theta} \tag{14}$$

□

Remark 1. *This bound can be rewritten in terms of the log-likelihood of the spontaneous jump and constants independent of the initial sequence x as:*

$$|\log Pr(x \rightarrow y) - C_0| \leqq C_1\theta \tag{15}$$

where the constants are given by:

$$C_0 = \log \omega_y^Q \tag{16}$$

$$C_1 = \frac{\sqrt{8}N^2}{1-\alpha} \tag{17}$$

Handling Missing Data & Curated Poles

The Q-net construction naturally handles missing entries during the construction of the component decision trees. Additionally, we can compute the q-distance between partially complete opinion vectors without any additional modification. This follows from the fact that if all responses $x_{-i} = \{x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{n-1}, x_n\}$ other than that at the index i is available, Φ_i is conditioned on x_{-i}, whereas if any other responses are missing, the distribution Φ_i is simply conditioned on a smaller set. This allows us to choose a smaller set of GSS variables, and enumerate responses which would reflect certain ideological leanings. These are referred to as "polar vectors", $e.g.$, the liberal pole, and the conservative pole.

The liberal and conservative poles ($\wp_\star, \wp^\star$ respectively) used in this study are shown in Table I. The poles corresponding to opinions on race-related items ($\nu^\star, \nu_\star$ respectively) and gender-related items ($\mu^\star, \mu_\star$ respectively) are shown in Extended Data Table VII.

Neither the choice of query items in defining these race/gender and ideological poles is not unique, nor the responses chosen to reflect the extreme ideological positions is unique. Nevertheless, we verified that different choices to reflect these vectors do not substantially change our results or conclusions.

Ideological Polarization Measures

Definition 7 (Polar Separation). *As a function of the inferred Q-net Φ^P for a population P, and the given poles $\wp_\star, \wp^\star$ (See Table I), the polar separation is defined as:*

$$d^\star \triangleq \theta_{\Phi^P}(\wp_\star, \wp^\star) \tag{18}$$

Definition 8 (Ideology index). *For a bipolar society, the ideology index of an opinion vector s for a population P with an inferred Q-net is defined as:*

$$\eta^P(s) = \frac{\theta_{\Phi^P}(s, \wp_\star) - \theta_{\Phi^P}(s, \wp^\star)}{\theta(\wp_\star, \wp^\star)} \tag{19}$$

where $\wp^\star, \wp_\star$ are the two polar vectors. In general for a multi-pole society, the ideology index measures the closeness to one of the poles. Thus, in general, the ideology index is a real-valued vector, where the i^{th} component is given by:

$$\eta^P(s)_i = \frac{\theta_{\Phi^P}(s, \wp^i) - \max_{j\neq i}\theta_{\Phi^P}(s, \wp^j)}{\max_{j\neq i}\theta_{\Phi^P}(\wp^i, \wp^j)} \tag{20}$$

Definition 9 (Embedding diameter). *The embedding diameter of a population P with an inferred Q-net is defined as:*

$$d_P \triangleq \mathbf{E}_{P'\sim P} \max_{x\in P', y\in P'} \theta_{\Phi^P}(x, y) \tag{21}$$

where P' is a sufficiently large sample from the population P.

Affective Polarization Measures

We formalize two measures of affective polarization measures, 1) the optimal number of clusters ($\mathcal{C}$) in year-specific metric embedding, where we optimize the number of clusters using the Bayesian Information criterion (BIC)[36], and 2) the average cluster separation ($d_\mathcal{C}$), which is the average distance in the metric embedding when we use the BIC-optimal number of clusters.

Race and Gender Related Indexes

Definition 10 (Race-related Index). *The race-related index is defined for an opinion vector s from a population P is defined as:*

$$R^P(s) = \frac{\theta_{\Phi^P}(\mu^\star, s) - \theta_{\Phi^P}(\mu_\star, s)}{\theta_{\Phi^P}(\mu^\star, \mu_\star)} \tag{22}$$

Definition 11 (Sexism Index). *The gender-related index is defined for an opinion vector s from a population P is defined as:*

$$S^P(s) = \frac{\theta_{\Phi^P}(\nu^\star, s) - \theta_{\Phi^P}(\nu_\star, s)}{\theta_{\Phi^P}(\nu^\star, \nu_\star)} \tag{23}$$

If the population is fixed $e.g.$ if we are considering a single year with a corresponding Q-net, the denominators can be ignored in the definition of the race and gender indices, as we do in our second validation exercise (prediction of 2016 US Presidential election outcome).

Reconstruction Approach: Q-sampling

The inferred Q-net can be used to optimally impute missing entries in an opinion vector, factoring in the constraints that the remaining known responses to the rest of the opinion vector confers. Given a population P inducing a Q-net Φ^P, we can sample the neighborhood of an opinion vector x, factoring in the inferred dependencies captured by Φ^p. This induces a random field $\mathbb{N}(x, \Phi^P, \mu)$ taking values in $\prod_{i=1}^N \Sigma_i$. A specific realization for $\mathbb{N}(x, \Phi^P, \mu) = \zeta(x, \Phi^P, \mu)$ is computed as shown in Algo. 1. Note that $\zeta(x, \Phi^P, \mu)$ is a random function of its inputs, and can potentially change each time it is evaluated, and as described above, and outputs a realization of the random field $\mathbb{N}(x, \Phi^P, \mu)$. We call ζ the *Q-sampler.*

Definition 12 (Q-sampling). *Given a population P inducing a Q-net Φ^P, we can sample the neighborhood of an opinion vector x via the Q-sampling algorithm, denoted by $\zeta(x, \Phi^P, \mu)$ (See Algo. 1). Here μ denotes a baseline or average probability of the i^{th} item getting perturbed by random chance, which is estimated as scaling with* *the variance of responses observed for that item in the overall population.*

Thus, the reconstruction approach used in this study may be described as follows: Let x be a partial opinion vector, with a missing response at index $i_\star \in \mathbb{I}$. We carry out q-sampling as follows:

$$x \leftarrow \zeta(x, \Phi^P, \mu) \tag{24}$$

stopping when $i_\star$ has been populated.

The q-sampling algorithm realizes a probabilistic dynamical system:

$$x \rightarrow \zeta(x, \Phi^P, \mu) \rightarrow \zeta(\zeta(x, \Phi^P, \mu), \Phi^P, \mu) \rightarrow \cdots \tag{25}$$

which also induces a (deterministic) dynamical system if we opt for a maximum likelihood choice for the perturbations, $i.e.$, if

$$\overline{\zeta}(x, \Phi^P, \mu) \triangleq \operatorname*{argmax}_{\sigma \in \Sigma_i} \Phi^P(x_{-i})|_\sigma \tag{26}$$

Note, that such a dynamical system has fixed points $x^\star$ defined by:

$$x^\star|_i = \operatorname*{argmax}_{\sigma \in \Sigma_i} \Phi^P(x_{-i})|_\sigma \tag{27}$$

which can be interpreted as the stable thought centers in society.

In general, q-sampling is a means to sample the joint distribution of responses to the set of query items in the survey. Note that the Q-net is a means of estimating conditional distributions, and a direct sampling of the ultra high dimensional joint distribution is intractable both computationally, and due to the impractical sample complexity required for such an approach. However, we can easily show that the q-sampling approach indeed samples from this joint distribution asymptotically.

Theorem 2 (Convergence of q-sampling)**.** *The q-sampling algorithm described in Algo. 1 samples the joint distribution of the survey items.*

Proof. The q-sampling algorithm is identical to Gibb's sampling[37], which has the required property. □

Algorithm 1: Q-sampling ($\zeta(x, \Phi^P, \mu)$)

Data: Baseline propapaility μ, Qnet Φ^P, opinion vector x
Result: opinion vector x'
/* choose item index to perturb */
Choose i with probability μ_i;
/* choose new response for item at index */
Choose $\sigma \in \Sigma_i$ with probability $\Phi^P_i|_\sigma$;
/* update opinion vector */
$x_i \leftarrow \sigma$;
return x;

Analysis of Causal Drivers of Societal Polarization

We consider GDP (g_D), GNP (g_N) , GNI ((g_I), trade balance (g_T), poverty index (g_P), US population, S&P 500 yearly close prices (g_M) and political representation in the US Congress (g_C) and the Senate (g_S) as putative factors driving societal polarization (See Fig. 4f). We fit multi-variable regression models to the polarization measures estimated over time choosing the regression equations via minimizing AIC over a large set of randomly generated equations, which led to the optimal regression equations:

$$\mathcal{C} = g_D + g_P + g_T + g_N + g_M + g_S g_C \tag{28}$$

$$d_C = g_D + g_P + g_T + g_N + g_M + g_S g_C \tag{29}$$

$$d_P = g_D + g_P + g_T + g_N + g_M + g_S g_C + d_C \tag{30}$$

$$d^\star = g_D + g_P + g_T + g_N + g_M + g_S g_C + d_C \tag{31}$$

Importantly, not all of the putative factors show up in the optimized equations, and not all factors included turn out to be significant (See Fig. 4d).

We also carried out pairwise Granger tests on all combinations of economic/political (9) and polarization variables (4), $i.e.$ in total $4 \times 9 \times 2 = 72$ such possible relations were tested, out of which seven directed links turn out to be statistically significant (Fig. 4e). We allowed for a range of time delays or lags in the Granger tests to ascertain if the relationships turn out to be significant with some time lag measured in the unit of years. Two out of the four significant relationships from affective to ideological polarization showed up with a lag of three years.

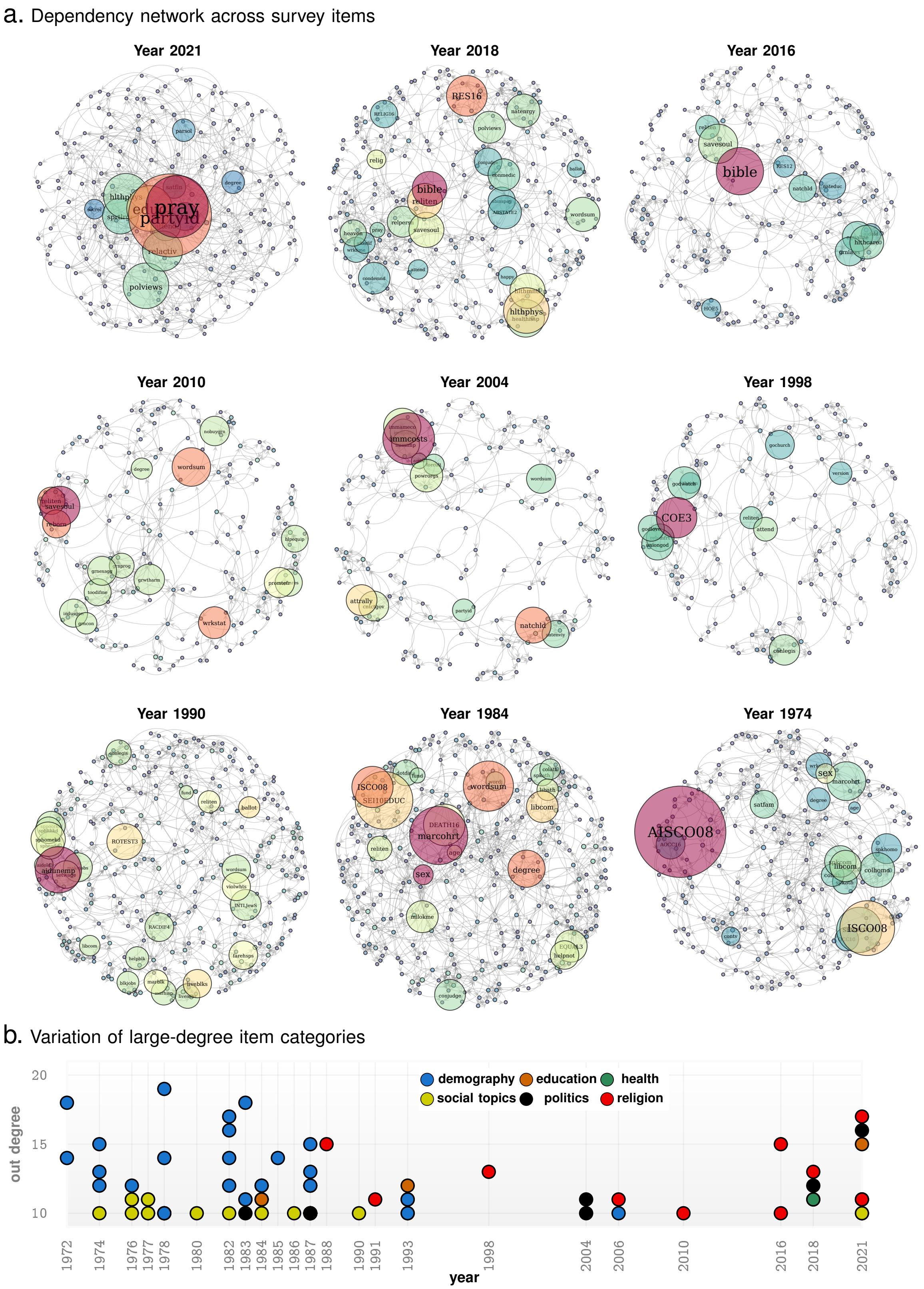
While correlation is not a measure of statistical causality, computing the Pearson's correlation matrix (Fig. 4f) revealed the strong association between the economic/political putative drivers and the measures of polarization defined in this study.

DATA & SOFTWARE AVAILABILITY

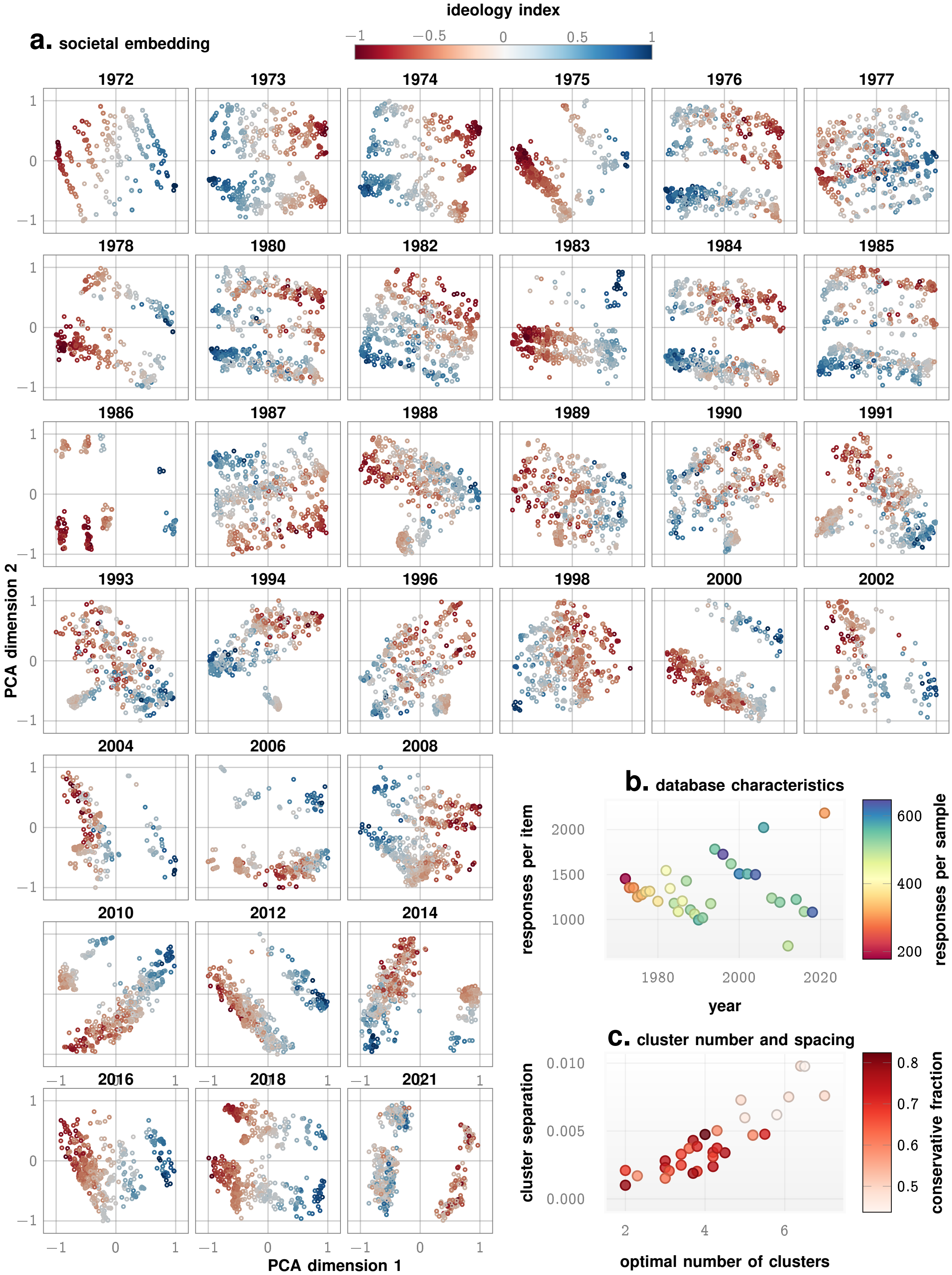
The GSS database is publicly accessible. The complete implementation for the CogNet architecture is available under a permissive license at https://pypi.org/project/cognet/, with complete instructions for installation in any Python 3.x environment. The inferred Q-net models are available at https://zenodo.org/record/5781768/.

ACKNOWLEDGMENT

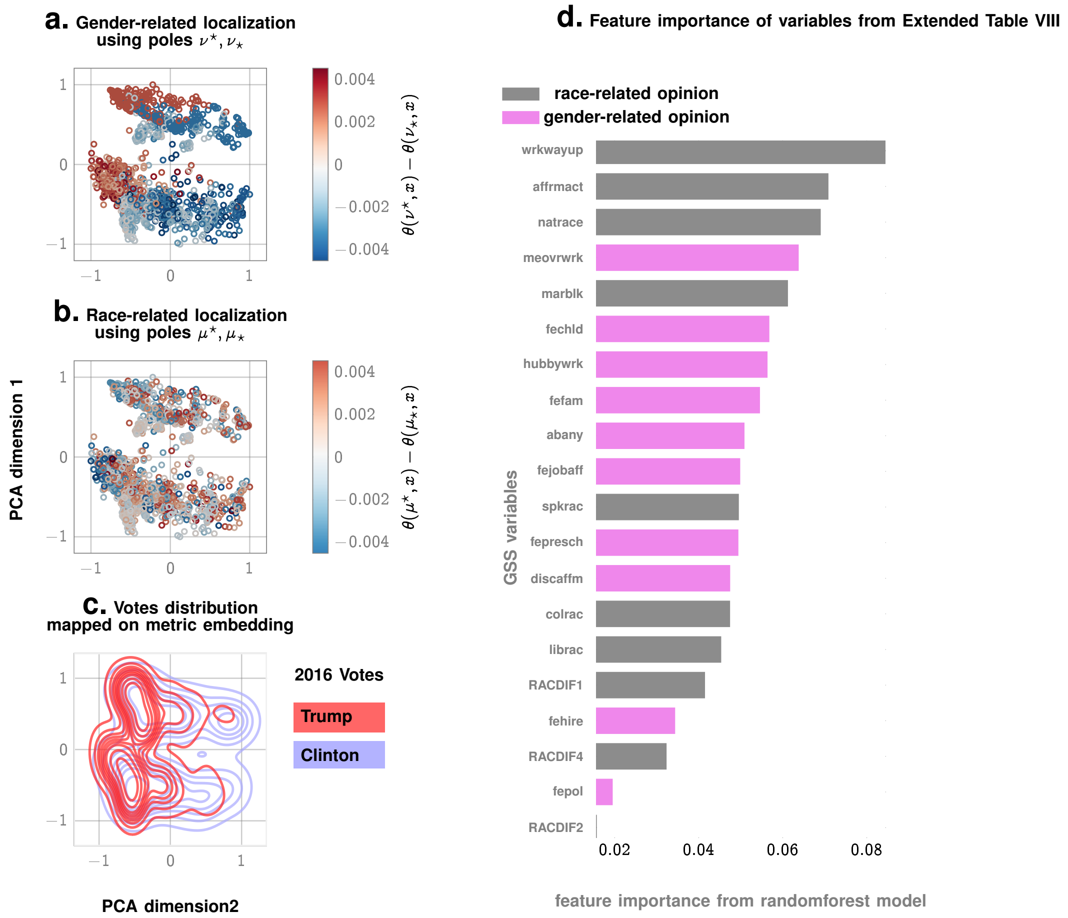
This work is funded in part by the Defense Sciences Office of the Defense Advanced Research Projects Agency (Project No. XX). The claims made in this study do not necessarily reflect the position or the policy of the sponsors, and no official endorsement should be inferred.



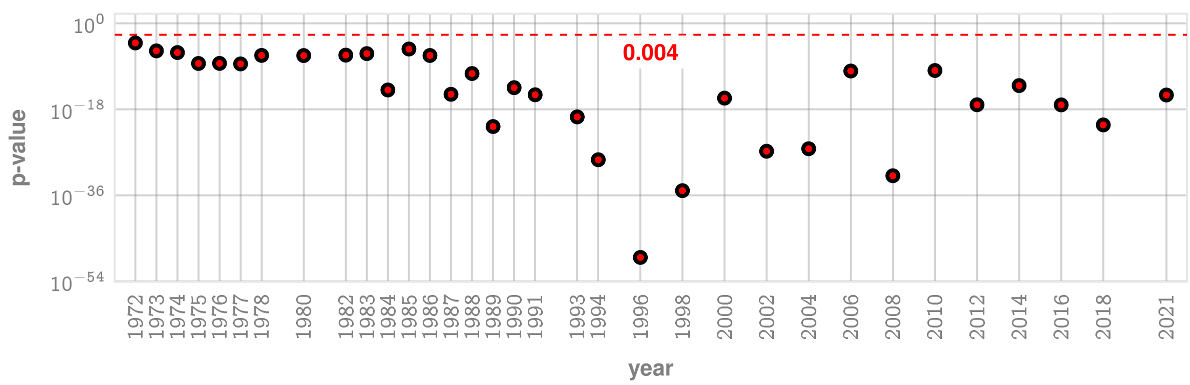
Extended Data Fig. 1. **Emergent dependency graphs for selected time-periods.** The Q-net models inferred with the GSS reponse data of each year results in a forest of conditional inference trees, which induces year-specific dependency graphs shown here. A directed edge from GSS variable A to varaibel B implies that teh conditional inference tree predicting B uses A as a feature. The nodes in the graphs shown here are therefore GSS variables, and the size and color scaling of the nodes represent the out-degree of the nodes: the larger a node, higher is the number of variables that the variable corresponding to this node affects significantly. Panel b illustrates that the composition of the set of high out-degree variables changes over time, with demographic chracterization of the participants and opinions on social topics replaced with religious beliefs and political opinions in more recent times. We also note that the maximum out-degree of the nodes seem to somewhat relfect the state of societal polarization, which we infer to be low in late 90s to early 2000s.



Extended Data Fig. 2. **Visualization of societal embedding via q-distance.** Panel a. The year-specific distance matrices obtained by computing pairwise q-distance between individual participants in the GSS surveys are mapped into a 2D plane using first a Sippl embedding (converting a distance matrix to a minimal-erroe high dimensional embedding), followed by a PCA construction (mapping a high dimensional emedding to an approximate 2D embedding). Each data point coresponds to an individual participant, and represents their opinion vector in this embedding. The color scale corresponds to the ideology index of their opinion vector, which ranges from red (conservative, closer to the conservative pole, which has an ideology index of -1) to blue (liberal, closer to the liberal pole, which has an ideology index of 1). Panel b shows that the embedding produces distinct clusters, and the average spacing between clusters seem to linearly increase with the optimal number (optimized via minimizing the Bayesian Information Criterion (BIC)) of cluster in any year.



Extended Data Fig. 3. **Additional data on Presidential election forecast for 2016.** Panel a shows the distribution of the defined index of sexism, mapped onto the metric embedding. Panel b shows teh corresponding figure with the racism index. Panel c shows the distribution of teh actual reported votes for each candidate. Note that the Trump votes line up well with high sexism index. Panel d shows the relative feature importances in a random forest model trained on raw survey data, also reflecting the importance of some particular beliefs in driving election votes.



Extended Data Fig. 4. The null hypothesis that a random selection of responses can generate the observed responses is rejected in every year with p-values as shown above. Here we use the straightforward normal approximation to the multinomial distribution of a varying number of possible responses to teh query items estimate the p-value.

Extended Data Table I
RECONSTRUCTION EXAMPLE (2018, SUBJECT NUMBER 706)

item	description	true	reconstructed
abany	abortion should be legal if mother wants it for any reason	yes	yes
abdefect	abortion should be legal if there is a strong chance of serious defect in baby	yes	yes
abpoor	abortion should be legal if family has a very low income and cannot afford any more children	yes	yes
abpoorw	wrong for woman to get abortion if low income	not wrong at all	not wrong at all
abrape	abortion should be legal if mother pregnant by rape	yes	no
absingle	abortion should be legal if mother is not married and does not want to marry the man	yes	yes
colcom	communist allowed to teach in a college	not fired	fired
colmil	militarists be allowed to teach in a college or university	allowed	allowed
godchnge	which best describes your beliefs about god	believe now, always have	believe now, didn't used to
gunlaw	require a person to obtain a police permit before he or she could buy a gun	favor	favor
intmil	interest in issues about military and defense policy	moderately interested	moderately interested
libhomo	book in favor of homosexuality allowed in public library	not remove	not remove
owngun	have in your home any guns or revolvers	no	no
pillok	birth control should be available to teenagers between the agesof 14 and 16 if their parents do not approve	strongly agree	strongly agree
prayer	Bible prayer in public schools	disapprove	disapprove
prayfreq	about how often do you pray	about once a month	every week
religcon	religions bring more conflict than peace	not agree/dsagre	agree
religint	people with very strong religious beliefs are often too intolerant of others	not agree/dsagre	not agree/dsagre
shotgun	own a shotgun	no	no
spkmil	militarists allowed to make a speech in your community	allowed	allowed

Extended Data Table II
RECONSTRUCTION EXAMPLE (2018, SUBJECT NUMBER 1040)

item	description	true	reconstructed
abany	abortion should be legal if mother wants it for any reason	yes	yes
abdefect	abortion should be legal if there is a strong chance of serious defect in baby	yes	yes
abnomore	abortion legal if mother does not want any more children	no	no
abpoor	abortion should be legal if family has a very low income and cannot afford any more children	no	no
abpoorw	wrong for woman to get abortion if low income	always wrong	not wrong at all
abrape	abortion should be legal if mother pregnant by rape	yes	yes
absingle	abortion should be legal if mother is not married and does not want to marry the man	no	yes
colmil	militarists be allowed to teach in a college or university	allowed	allowed
comfort	practicing a religion helps people to gain comfort in times of trouble and sorrow	agree	agree
godchnge	which best describes your beliefs about god	believe now, always have	believe now, always have
gunlaw	require a person to obtain a police permit before he or she could buy a gun	favor	favor
libhomo	book in favor of homosexuality allowed in public library	not remove	not remove
libmil	allow militarists book in library	not remove	not remove
libmslm	allow anti-american muslim clergymen's books in library	not remove	not remove
owngun	have in your home any guns or revolvers	yes	no
pillok	birth control should be available to teenagers between the agesof 14 and 16 if their parents do not approve	strongly agree	strongly disagree
prayfreq	about how often do you pray	several times a week	several times a week
rowngun	own a gun	no	no
shotgun	own a shotgun	yes	no
spkcom	communist allowed to make a speech in your community	allowed	allowed

Extended Data Table III
RECONSTRUCTION EXAMPLE (2018, SUBJECT NUMBER 1297)

item	description	true	reconstructed
abany	abortion should be legal if mother wants it for any reason	no	yes
abhlth	abortion should be legal if mother's own health is seriously endangered by the pregnancy	yes	no
abnomore	abortion legal if mother does not want any more children	no	no
abpoor	abortion should be legal if family has a very low income and cannot afford any more children	no	no
abpoorw	wrong for woman to get abortion if low income	always wrong	always wrong
abrape	abortion should be legal if mother pregnant by rape	no	no
colcom	communist allowed to teach in a college	not fired	not fired
godchnge	which best describes your beliefs about god	believe now, always have	believe now, always have
gunlaw	require a person to obtain a police permit before he or she could buy a gun	favor	favor
libcom	communist books allowed in your public library	not remove	not remove
libmil	allow militarists book in library	not remove	not remove
pillok	birth control should be available to teenagers between the agesof 14 and 16 if their parents do not approve	strongly disagree	disagree
pray	about how often do you pray	once a day	several times a day
prayer	Bible prayer in public schools	disapprove	approve
prayfreq	about how often do you pray	every week	several times a week
religcon	religions bring more conflict than peace	disagree	disagree
religint	people with very strong religious beliefs are often too intolerant of others	disagree	agree
reliten	would you call yourself a strong religious person	strong	strong
shotgun	own a shotgun	no	no
spkmil	militarists allowed to make a speech in your community	allowed	allowed

Extended Data Table IV
RECONSTRUCTION EXAMPLE (2016, SUBJECT NUMBER 354)

item	description	true	reconstructed
abdefect	abortion should be legal if there is a strong chance of serious defect in baby	yes	yes
abhlth	abortion should be legal if mother's own health is seriously endangered by the pregnancy	yes	yes
abnomore	abortion legal if mother does not want any more children	yes	yes
abpoor	abortion should be legal if family has a very low income and cannot afford any more children	yes	yes
abrape	abortion should be legal if mother pregnant by rape	yes	yes
bible	the bible is the actual word of god and is to be taken literally or is a book of fables	inspired word	word of god
grass	use of marijuana should be made legal	legal	not legal
gunlaw	require a person to obtain a police permit before he or she could buy a gun	favor	favor
libcom	communist books allowed in your public library	not remove	not remove
libhomo	book in favor of homosexuality allowed in public library	not remove	not remove
libmil	allow militarists book in library	not remove	not remove
libmslm	allow anti-american muslim clergymen's books in library	not remove	remove
owngun	have in your home any guns or revolvers	yes	no
polabuse	policeman can strike a citizen who says vulgar and obscene things to the policeman	no	no
rowngun	own a gun	yes	yes
shotgun	own a shotgun	no	no
spkcom	communist allowed to make a speech in your community	allowed	allowed
spkmil	militarists allowed to make a speech in your community	allowed	allowed

Extended Data Table V
RECONSTRUCTION EXAMPLE (2008, SUBJECT NUMBER 1909)

item	description	true	reconstructed
abdefctw	abortion is wrong if there is a strong chance of serious defect in the baby	always wrong	always wrong
abhlth	abortion should be legal if mother's own health is seriously endangered by the pregnancy	yes	yes
abnomore	abortion legal if mother does not want any more children	no	no
abpoor	abortion should be legal if family has a very low income and cannot afford any more children	no	no
absingle	abortion should be legal if mother is not married and does not want to marry the man	no	no
bible	the bible is the actual word of god and is to be taken literally or is a book of fables	word of god	word of god
colcom	communist allowed to teach in a college	not fired	not fired
colmil	militarists be allowed to teach in a college or university	allowed	allowed
conlabor	confidence in organized labor	only some	only some
godchnge	which best describes your beliefs about god	believe now, always have	believe now, always have
grass	use of marijuana should be made legal	not legal	legal
gunlaw	require a person to obtain a police permit before he or she could buy a gun	favor	oppose
libcom	communist books allowed in your public library	remove	not remove
libhomo	book in favor of homosexuality allowed in public library	not remove	remove
libmil	allow militarists book in library	not remove	not remove
libmslm	allow anti-american muslim clergymen's books in library	not remove	remove
polabuse	policeman can strike a citizen who says vulgar and obscene things to the policeman	no	no
pray	about how often do you pray	several times a day	several times a day
prayfreq	about how often do you pray	several times a day	several times a day
shotgun	own a shotgun	no	no
spkcom	communist allowed to make a speech in your community	allowed	allowed
taxrich	describe taxes in america today	too high	much too high

Extended Data Table VI
RECONSTRUCTION EXAMPLE WITH POOR RECONSTRUCTION PERFORMANCE (2008, SUBJECT NUMBER 1076)*

item	description	true	reconstructed
abany	abortion should be legal if mother wants it for any reason	yes	yes
abdefctw	abortion is wrong if there is a strong chance of serious defect in the baby	not wrong at all	wrong only sometimes
abdefect	abortion should be legal if there is a strong chance of serious defect in baby	yes	yes
abnomore	abortion legal if mother does not want any more children	yes	yes
abpoor	abortion should be legal if family has a very low income and cannot afford any more children	yes	yes
abpoorw	wrong for woman to get abortion if low income	always wrong	almost always wrong
absingle	abortion should be legal if mother is not married and does not want to marry the man	yes	yes
bible	the bible is the actual word of god and is to be taken literally or is a book of fables	inspired word	inspired word
colcom	communist allowed to teach in a college	not fired	fired
colmil	militarists be allowed to teach in a college or university	not allowed	not allowed
comfort	practicing a religion helps people to gain comfort in times of trouble and sorrow	strongly agree	agree
conlabor	confidence in organized labor	hardly any	hardly any
grass	use of marijuana should be made legal	not legal	not legal
libcom	communist books allowed in your public library	not remove	not remove
libmil	allow militarists book in library	remove	not remove
owngun	have in your home any guns or revolvers	no	yes
polabuse	policeman can strike a citizen who says vulgar and obscene things to the policeman	no	yes
pray	about how often do you pray	lt once a week	several times a day
religint	people with very strong religious beliefs are often too intolerant of others	agree	strongly agree
reliten	would you call yourself a strong religious person	not very strong	strong
shotgun	own a shotgun	no	yes
spkcom	communist allowed to make a speech in your community	allowed	not allowed
spkmil	militarists allowed to make a speech in your community	allowed	not allowed
taxrich	describe taxes in america today	too high	about right

* Probability of getting worse reconstruction is less than 12%

Extended Data Table VII
POLAR VECTORS USED TO IDENTIFY IDEOLOGIVAL LEANING TO RACE/GENDER INSENSITIVITY OR VIEWS THAT FAIL TO PROMOTE SOCIAL EQUALITY

index	description	less sexism/racism	more sexism/racism	type
		$\mu^{\star}$	$\mu_{\star}$	
RACDIF1	Blacks have worse jobs income etc than white people due to discrimination	yes	no	race
RACDIF2	Because most Blacks have less inborn ability to learn	no	yes	race
RACDIF4	Because most Blacks just donot have sufficient motivation	no	yes	race
natrace	govt spending on improving the conditions of Blacks	too little	too much	race
colrac	Racists allowed to teach in a college or university	not allowed	allowed	race
affrmact	Oppose affirmative action	support pref	oppose pref	race
wrkwayup	Blacks should work their way up without special favors	agree strongly	disagree strongly	race
librac	Book written by racists be taken out of your public library	remove	not remove	race
spkrac	Racists allowed to speak in community speeches	not allowed	allowed	race
marblk	Comfortable having a close relative marry a black person	favor	oppose	race
		$\nu^{\star}$	$\nu_{\star}$	
fepol	Most men are better suited emotionally for politics than are most women	disagree	agree	gender
fechld	A working mother can establish just as warm and secure a relationship with her children	agree	disagree	gender
fepresch	A preschool child is likely to suffer if his or her mother works	disagree	agree	gender
fejobaff	Preferential hiring of women	for	against	gender
discaffm	Now equally or less qualified woman gets job or promotion instead of man	very unlikely	very likely	gender
fehire	Should hire and promote women	agree	disagree	gender
hubbywrk	A husband should earn money and a wife should look after family	disagree	agree	gender
meovrwrk	Family life often suffers because men concentrate too much on their work	agree	disagree	gender
abany	Abortion ok for any reason	yes	no	gender
fefam	Man should be the achiever outside while woman takes care of family	disagree	agree	gender

REFERENCES

[1] Smith, T. W., Davern, M., Freese, J. & Morgan, S. General social surveys, 1972-2016 [machine-readable data file]/principal investigator, smith, tom w.; co-principal investigators, peter v. marsden and michael hout; sponsored by national science foundation. *Chicago: NORC* (2018).
[2] Heltzel, G. & Laurin, K. Polarization in america: two possible futures. *Current opinion in behavioral sciences* **34**, 179–184 (2020).
[3] Fiorina, M. P. & Abrams, S. J. Political polarization in the american public. *Annu. Rev. Polit. Sci.* **11**, 563–588 (2008).
[4] Iyengar, S., Sood, G. & Lelkes, Y. Affect, not ideologya social identity perspective on polarization. *Public opinion quarterly* **76**, 405–431 (2012).
[5] Ura, J. D. & Ellis, C. R. Partisan moods: Polarization and the dynamics of mass party preferences. *The Journal of Politics* **74**, 277–291 (2012).
[6] Druckman, J. N., Peterson, E. & Slothuus, R. How elite partisan polarization affects public opinion formation. *American Political Science Review* **107**, 57–79 (2013).
[7] Grosser, J. & Palfrey, T. R. Candidate entry and political polarization: An antimedian voter theorem. *American Journal of Political Science* **58**, 127–143 (2014).
[8] Lauderdale, B. E. Does inattention to political debate explain the polarization gap between the us congress and public? *Public Opinion Quarterly* **77**, 2–23 (2013).
[9] Levendusky, M. S. Why do partisan media polarize viewers? *American Journal of Political Science* **57**, 611–623 (2013).
[10] Prior, M. The challenge of measuring media exposure: Reply to dilliplane, goldman, and mutz. *Political Communication* **30**, 620–634 (2013).
[11] Leeper, T. J. The informational basis for mass polarization. *Public Opinion Quarterly* **78**, 27–46 (2014).
[12] Thomsen, D. M. Ideological moderates won't run: How party fit matters for partisan polarization in congress. *The Journal of Politics* **76**, 786–797 (2014).
[13] Weinschenk, A. C. Polarization, ideology, and vote choice in us congressional elections. *Journal of Elections, Public Opinion & Parties* **24**, 73–89 (2014).
[14] Mason, L. "i disrespectfully agree": The differential effects of partisan sorting on social and issue polarization. *American Journal of Political Science* **59**, 128–145 (2015).
[15] Bramson, A. *et al.* Understanding polarization: Meanings, measures, and model evaluation. *Philosophy of science* **84**, 115–159 (2017).
[16] Bramson, A. *et al.* Disambiguation of social polarization concepts and measures. *The Journal of Mathematical Sociology* **40**, 80–111 (2016).
[17] DeGroot, M. H. Reaching a consensus. *Journal of the American Statistical Association* **69**, 118–121 (1974).
[18] Friedkin, N. E. & Johnsen, E. C. Social influence and opinions. *Journal of Mathematical Sociology* **15**, 193–206 (1990).
[19] Pariser, E. *The filter bubble: What the Internet is hiding from you* (Penguin UK, 2011).
[20] Bakshy, E., Messing, S. & Adamic, L. A. Exposure to ideologically diverse news and opinion on facebook. *Science* **348**, 1130–1132 (2015).
[21] Musco, C., Musco, C. & Tsourakakis, C. E. Minimizing polarization and disagreement in social networks. In *Proceedings of the 2018 World Wide Web Conference*, 369–378 (2018).
[22] Mao, Y., Bolouki, S. & Akyol, E. Spread of information with confirmation bias in cyber-social networks. *IEEE Transactions on Network Science and Engineering* **7**, 688–700 (2018).
[23] Aslay, C., Matakos, A., Galbrun, E. & Gionis, A. Maximizing the diversity of exposure in a social network. In *2018 IEEE International Conference on Data Mining (ICDM)*, 863–868 (IEEE, 2018).
[24] Bail, C. A. *et al.* Exposure to opposing views on social media can increase political polarization. *Proceedings of the National Academy of Sciences* **115**, 9216–9221 (2018).
[25] Sard-Espinosa, A., Subbiah, S. & Bartz-Beielstein, T. Conditional inference trees for knowledge extraction from motor health condition data. *Eng. Appl. Artif. Intell.* **62**, 26–37 (2017). URL https://doi.org/10.1016/j.engappai.2017.03.008.
[26] Granger, C. Investigating causal relations by econometric models and cross-spectral methods. In *Essays in econometrics: collected papers of Clive WJ Granger*, 31–47 (2001).
[27] Bonikowski, B. Ethno-nationalist populism and the mobilization of collective resentment. *The British journal of sociology* **68**, S181–S213 (2017).
[28] Hawkins, K. & Littvay, L. *Contemporary US populism in comparative perspective* (Cambridge University Press, 2019).
[29] Steven, L. & Daniel, Z. How democracies die. *United States: Crown* (2018).
[30] Stewart, A. J., McCarty, N. & Bryson, J. J. Polarization under rising inequality and economic decline. *Science advances* **6**, eabd4201 (2020).
[31] Gidron, N., Adams, J. & Horne, W. *American affective polarization in comparative perspective* (Cambridge University Press, 2020).

[32] Wagner, M. Affective polarization in multiparty systems. *Electoral Studies* **69**, 102199 (2021).
[33] Manning, C. D., Manning, C. D. & Schütze, H. *Foundations of statistical natural language processing* (MIT press, 1999).
[34] Cover, T. M. *Elements of information theory* (John Wiley & Sons, 1999).
[35] Fedotov, A. A., Harremoës, P. & Topsoe, F. Refinements of pinsker's inequality. *IEEE Transactions on Information Theory* **49**, 1491–1498 (2003).
[36] Neath, A. A. & Cavanaugh, J. E. The bayesian information criterion: background, derivation, and applications. *Wiley Interdisciplinary Reviews: Computational Statistics* **4**, 199–203 (2012).
[37] Casella, G. & George, E. I. Explaining the gibbs sampler. *The American Statistician* **46**, 167–174 (1992).